Spring 2014 **Math 601** Name: ______________________

Quiz 3

Question 1. (12 pts)

Determine if the given subset is a subspace of the corresponding vector space. (**Show work!**).

(a) (4 pts) The subset of $\mathbb{R}^3$:

$$W = \{v \in \mathbb{R}^3 \mid v \cdot (1, 2, 3) = 0\}$$

Solution: Yes, W is a subspace. I leave it to you to verify the conditions.

(b) (4 pts) Let $\mathcal{M}_{n\times n}$ be the vector space of all real $n \times n$ matrices.

$$W = \{A \in \mathcal{M}_{n\times n} \mid A^T = A\}$$

Solution: Yes, W is a subspace. I leave it to you to verify the conditions.

(c) (4 pts) Let $P_n(t)$ be the vector space of all polynomials with degree $\leq n$.

$$W = \{p \in P_n(t) \mid p(1) = 2\}$$

Solution: Notice that the zero polynomial is not in W. So W is not a subspace.

Question 2. (8 pts)

Let $u_1 = (1,1,1)$, $u_2 = (1,2,3)$ and $u_3 = (1,5,8)$ in $\mathbb{R}^3$. Determine whether $v = (3,0,1)$ is a linear combination of $\{u_1, u_2, u_3\}$.

Solution: need to solve the linear system

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 5 & 0 \\ 1 & 3 & 8 & 1 \end{array}\right]$$

Use Gaussian elimination and we get

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

So $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \\ 4 \end{bmatrix}$ is a solution. So v is a linear combination of $\{u_1, u_2, u_3\}$.